FORTHPUTTING THE CONCEPT OF MAXWELL'S ETHER FOR THE DETERMINATION OF DIELECTRIC AND MAGNETIC FUNCTIONS OF A PHOTONIC CRYSTAL FROM THE FREQUENCY SPECTRUM

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Abstract

A new method is proposed for the determination of the effective dielectric permittivity and magnetic permeability of a photonic crystal in a narrow frequency range near the spectral singularities at the center of the Brillouin zone. Additional conditions for the applicability of the method are discussed.

1. Introduction

Usually it is considered that the effective refractive index of a material should satisfy the condition $Re(n) \geq 0$ [1, 2]. It is considered that the effective magnetic function $\mu=1$, while the dielectric function ε can take any values. The common procedure for the determination of μ_{eff} and ε_{eff} values relies on the consideration of the magnetic and electric momenta of a medium [1, 2].

Recently, it was shown that for some materials in a narrow frequency interval, including the optical spectral region, the following conditions can be fulfilled: $n_{eff} <0$ and $\mu_{eff} <0$. Notomi proposed a method for the determination of the effective refractive index n_{eff} of a photonic crystal by using its frequency spectrum [3]. However, Notomi noted that the result may have a formal meaning, and it is necessary to take into consideration additional factors which can influence the final result. Apart from the spectrum, the eigenvalue functions corresponding to the spectrum are of especial importance. Their influence may be decisive. The method proposed by Notomi has several drawbacks: (i) it allows only the determination of the effective refractive index n_{eff}, rather than the magnetic μ_{eff} and dielectric ε_{eff} functions; (ii) it does not indicate how the branch of the spectrum should be selected (when there are several branches). Other methods have been proposed for the determination of the effective functions [4, 5]. However, these methods are rather complicated.

In this work we propose a new method for the determination of the effective refractive index $n_{eff}(\omega)$ and the magnetic $\mu_{eff}(\omega)$ and dielectric $\varepsilon_{eff}(\omega)$ functions, as well as procedures which demonstrate the applicability of the method on the basis of the photonic crystal spectrum. It is supposed that, in a narrow frequency interval, the photonic crystal obeys the Maxwell equations and changes its frequency spectrum as compared to the vacuum, on the one hand, and it exhibits the properties of a homogeneous optical material with effective parameters, on the other hand. The method relies on the investigation of the influence of a hypothetic medium like the Maxwell's ether on the photonic crystal spectrum. The "ether" is characterized by a given magnetic $\delta\mu(\omega)$ (or dielectric $\delta\varepsilon(\omega)$) function, and all the space, including the material bodies, is filled with this "ether". This approach can be implemented through numerical modeling with a computer. Similarly to the method proposed by Notomi, this approach makes it possible to determine the values of effective parameters, which may have a formal character. Additional experimental and calculation procedure are necessary to demonstrate that these values are real ones, as described in the next section.

2. The method with the Ether

We will demonstrate the method using the example of a two-dimensional photonic crystal placed in an electromagnet field polarized perpendicular to the plane. We suppose that the spectrum has at least one singularity with one or two branches. The case with two branches is illustrated in Fig. 1a.

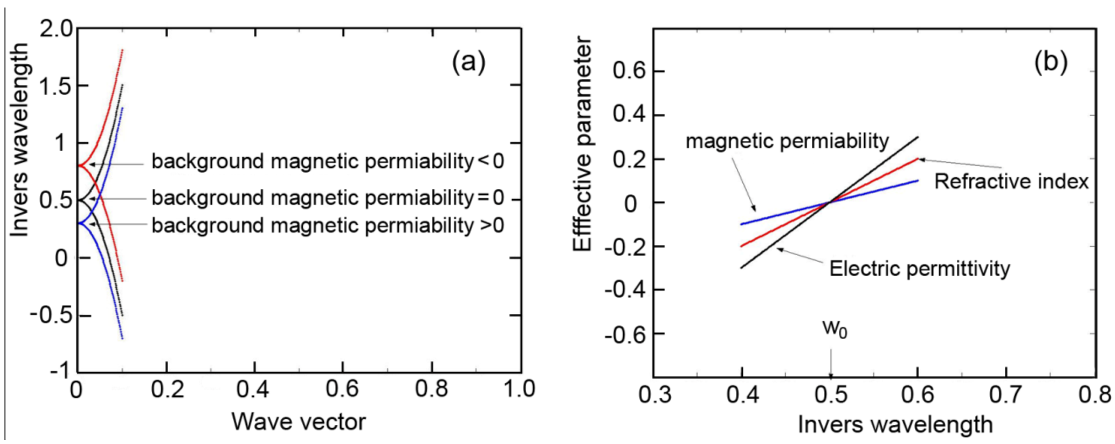

Fig. 1. (a) Dispersion curves at a singularity of a photonic crystal embedded in the "ether" with the background magnetic permeability. (b) Dependence of the effective parameters on the inverse wavelength.

We also suppose the following: (i) in a narrow spectral interval near the singularity $\omega = \omega_0$ at the center of the Brilloiun zone, the photonic crystal can be described as a homogeneous medium with the effective functions $n_{eff}(\mu_{eff}, \varepsilon_{eff})$ satisfying the equality

$$n_{eff} = \pm\sqrt{\mu_{eff}\varepsilon_{eff}}, \tag{1}$$

where the signs "+' or "-" are chosen according to the Veselago's rule [6];
(ii) the values of $n_{eff}(\mu_{eff}, \varepsilon_{eff})$ are functions of frequency, and they equal to zero simultaneously ($n_{eff}(\mu_{eff}, \varepsilon_{eff}) \equiv 0$) at $\omega = \omega_0$.

The method is illustrated in the following figures.

To determine the value of μ_{eff} at frequency ω, the space is filled with an "ether" with the magnetic permeability $\delta\mu(\omega)$. Then the total permeability introduced in the dispersion equation of the photonic crystal [6] equals to

$$\mu = 1+\delta\mu(\omega). \tag{2}$$

The value of $\delta\mu(\omega)$ is changed in a manner assuring the "shift" of the singularity from the frequency ω_0 to the frequency ω, for which the value of μ_{eff} should be determined. Then, the value of μ_{eff} will become equal to zero ($\mu_{eff} \equiv 0$), and it follows from the Maxwell equations [7,8] that

$$\mu\omega^2 = \omega_0^2. \tag{3}$$

On the other hand, we will consider that the photonic crystal behaves like a homogeneous material. Then, in order the homogeneous magnetic medium with μ_{eff} to acquire the magnetic permeability $\mu_{eff} \equiv 0$, it is necessary to fulfill the equality

$$\delta\mu(\omega) = -\mu_{eff} . \quad (4).$$

One can obtain from equations (2-4):

$$\mu_{eff} = 1 - \omega_o^2/\omega^2. \quad (5)$$

In order to determine ε_{eff}, one can use a similar "ether", but with the dielectric permittivity $\delta\varepsilon(\omega)$. However, in this case, it is impossible to obtain a simple analytical dependence, and it is necessary to perform numerical calculations for a given photonic crystal. Nevertheless, the qualitative peculiarities are the same. The values of $n_{eff}(\mu_{eff}, \varepsilon_{eff})$ in the neighborhood of ω_0 are described by the following expression:

$$n_{eff}(\omega) \sim (dn_{eff}/d\omega)_{\omega o}(\omega - \omega_0). \quad (6)$$

3. Demonstration of the method

Figure 2 presents the band structure of a two-dimensional photonic crystal and the effective functions $n_{eff}(\mu_{eff}, \varepsilon_{eff})$ for three singularity points ((1) $a/\lambda = 0.58$; (2) $a/\lambda = 0.72$; (3) $a/\lambda = 0.98$) near the center of the Brillouin zone calculated on the basis of this spectrum. The photonic crystal consisting of a square lattice of cylinders is placed in an electromagnetic field polarized perpendicular to the plane. The radius of cylinders is $r = 0.18a$ (a is the lattice constant), and the dielectric permittivity is $\varepsilon = 8.4$. The algorithm described in [8] is used. The inverse wavelength $a/\lambda = 2\pi\omega/c$ is used in the graph instead of the frequency.

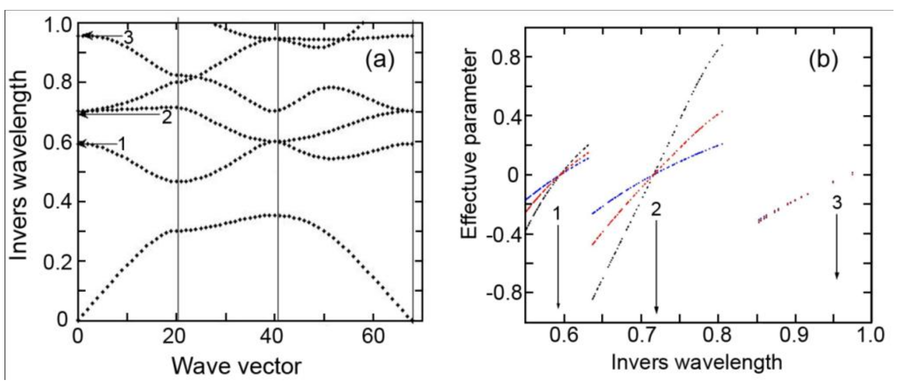

Fig. 2. (a) The band structure of a photonic crystal. (b) The effective parameters as a function of the inverse wavelength calculated for three singularity points.

Since the obtained effective parameters may have a mere formal character, one needs to apply additional experimental and calculations procedures to check their real meaning. For the beginning, we will compare the frequency spectrum with the frequency dependence of the transmission T or/and reflection R coefficients for two cases: (i) the crystal is modeled as a slab with the thickness h, (ii) the crystal represents a cylinder system arranged in a two-dimensional lattice with the same dimensions. If the concept of homogeneous medium is valid, then the crystal may be described as a medium with the effective optical parameters $n_{eff}(\mu_{eff}, \varepsilon_{eff})$, and we have

$$T = t_{13}(1 + r_{12}r_{23})/(e^{-i\Phi} + e^{i\Phi}r_{12}r_{23}), \quad (7)$$

$$R = (e^{-i\Phi}r_{12} + e^{i\Phi}r_{23})/(e^{-i\Phi} + e^{i\Phi}r_{12}r_{23}), \quad (8)$$

where T(R) are the transmission (reflection) coefficients of the slab, $r_{ij}(t_{ij})$ are the reflection (transmission) coefficients at the interfaces of i and j media. Φ is the difference of phases acquired during the moving of the wave through the slab. The value of r_{23} is determined as [1] $r_{23}= (r_{13} - r_{12})/(1 - r_{12}r_{13})$. If the slab is embedded in the vacuum, then $r_{12} = r_{23} = r$, $t_{13} = 1$.

The reflection coefficient at the plane boundary between the vacuum and the medium with effective parameters μ_{eff} and ε_{eff} equals

$$r = (1-Z_{eff})/(1+Z_{eff}), \tag{9}$$

where the impedance is $Z_{eff} = \sqrt{\mu_{eff}\,\varepsilon_{eff}}$.

We will consider only the transmittance T in the following discussions. The condition $n_{eff}<<1$ should be fulfilled near the singularity $\omega \approx \omega_o$. At the same time, the following equality will be valid

$$\Phi \approx (\omega h n_{eff}/c). \tag{10}$$

Then, taking into account (6), one obtains

$$T \approx 1- (\omega h|(dn_{eff} /d\omega)_{\omega o}(\omega -\omega_o)|/c)(1- r^2)/(1+ r^2). \tag{11}$$

As a result, a peak should be observed in the spectral dependence of the transmission coefficient near the singularities $\omega \approx \omega_o$, and the transmission should be extremely high (near 100%) at these points.

The transmission coefficient calculated by means of a multiple scattering approach (MSA) method [9] for a crystal composed of separate cylinders arranged in a square lattice is presented in Fig. 3.

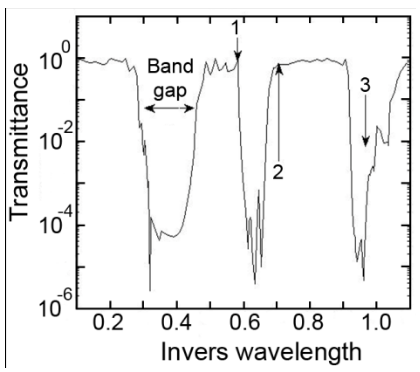

Fig. 3. Transmission coefficient for a photonic crystal composed of cylinders arranged in a square lattice.

One can see from the analysis of Fig. 3 that singularity point 3 does not satisfy the necessary conditions, since T<<1 at this point. Points 1 and 2 satisfy the condition better, and can be used for a further analysis. The graph should exhibit some pulsations near the singularities due to the interference of rays reflected from the front and rear surfaces of the slab. The amplitude of these pulsations should be equal to zero if $\mu_{eff} =\varepsilon_{eff}$, but is should be near 100% in the case of $\mu_{eff} /\varepsilon_{eff} >>1$ or $\mu_{eff}/ \varepsilon_{eff} <<1$. The amplitude of pulsations is near 20% in our case. That means that the deep minimum between points 1 and 2 is not due to the values of μ_{eff} and ε_{eff}, but is due to other reasons. Therefore, the regions where the effective parameters are well defined lie on left side of 0.59 and on the right side of 0.68 of the inverse wavelength values.

In order to choose between points 1 and 2, we will apply the following test with the ability to transform the wavefront from a point source to a plane wave. This property is inherent to media with ultra low refractive index [10, 11]. The results of this test for points 1 and 2 are shown in Fig. 4, which represents the distribution of the electric field perpendicular to the plane E(x,y), which makes it possible to implement the configuration of the wavefronts behind the slab. One can see that point 2 better satisfies this test, and it can be considered as a center of a frequency range for which the photonic crystal can be treated as a homogeneous medium with the properties described by formula (6). However, the poor results of this test for point 1 could be related to the interference effects of rays reflected from the front and rear surfaces of the slab, rather than the properties of the material constituting the slab. In fact, one can see from Fig. 3 that there are strong oscillations in the spectrum on the left side of point 1.

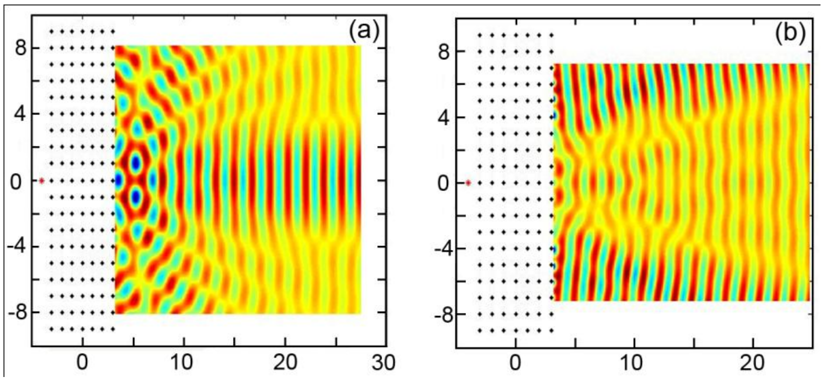

Fig. 4. Distribution of the electric field perpendicular to the plane E(x,y) calculated at inverse wavelength values of 0.58 (a) and 0.72 (b).

In order to additionally discriminate between the interference effects and the properties of the material system we applied a new test, which relies on the investigation of the superlensing effects. The superlensing effect is a clear demonstration of the presence of a material with a negative refractive index. For this purpose, we place the investigated slab in an "ether" with the dielectric permittivity

$$\Delta\varepsilon = -0.5(1+\varepsilon_{eff}) \tag{12}$$

and the magnetic permeability

$$\Delta\mu = -0.5(1+\mu_{eff}). \tag{13}$$

In this case, both the spaces occupied by the vacuum and the photonic crystal acquire equal values of μ_{eff} and ε_{eff}, but with different sign. The refractive index of the media is

$$n_{\Delta} = \sqrt{(1-\varepsilon_{eff})(1-\mu_{eff})} \tag{14}$$

but has a different sign.

Two factors could hinder the superlasing effect [12]: (i) the relative refractive index different from -1; (ii) the impedance of the crystal different from 1 which results in multiple reflections from the crystal/vacuum interface. In our case, we have the relative refractive index n = -1, and the relative impedance Z = 1. Therefore, we have ideal conditions for implementing the superlensing effect, and these conditions can be used for testing the applicability of the concept of effective refractive index for the investigated frequency (inverse wavelength) interval. We performed calculations for the inverse wavelength values of $a/\lambda = 0.58$ and $a/\lambda = 0.72$. Both the effective magnetic μ_{eff} and dielectric ε_{eff} functions are zero near this singularities. Then, $\Delta\varepsilon = \Delta\mu = -0.5$; $n_{\Delta} = 0.5$ for both values of the inverse wavelength. The results of simulation are shown in Fig. 5.

One can realize from this figure a clear superlensing effect for the inverse wavelength value of 0.72 corresponding to singularity point 2, and the absence of this effect for the inverse wavelength value of 0.57 corresponding to singularity point 1. The results of this test corroborate the results of the previous test. The emergence of additional reflexes in Fig. 5b is due to the spatial dispersion of the photonic crystal [13].

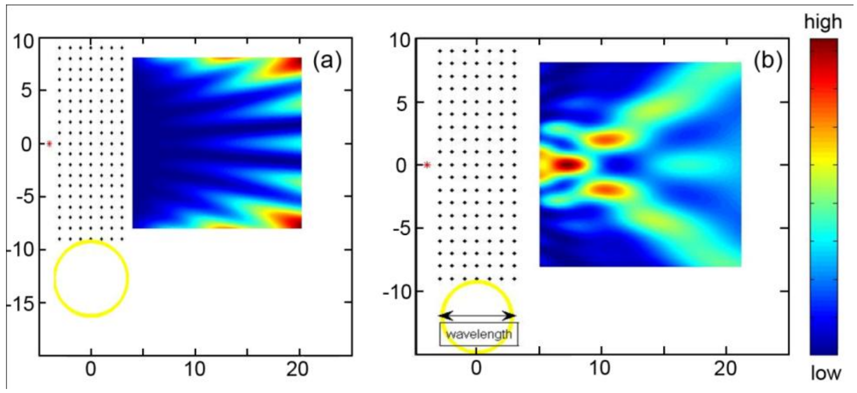

Fig. 5. Electric field intensity map of a cross-sectional view of the 2D source–image system when imaging by a photonic crystal slab consisting of a square lattice of cylinders calculated at inverse wavelength values of 0.58 (a) and 0.72 (b). The diameter of the circle in the figure corresponds to the wavelength.

4. Conclusions

A new method is proposed for the determination of the effective parameters of a photonic crystal based on the analysis of its frequency dispersion relations which avoid the work with complex equifrequency-surface (EFS) contours. The method makes it possible to determine simultaneously the three parameters μ_{eff}, ε_{eff}, and n_{eff} of the photonic crystal as well as their frequency dependence. The method is proposed for testing the applicability of the proposed approach in a real case on the basis of using an "ether" substratum.

Acknowledgments

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References

[1] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, Pergamon Press, Oxford, 1960.
[2] V. M. Agranovich and Yu. N. Gartstein, Usp. Fiz. Nauk 176, 1051, (2006).
[3] M. Notomi, Phys. Rev. B 62, 10696, (2000).
[4] T. Decoopman, G. Tayeb, S. Enoch, D. Maystre, and B. Gralak, Phys. Rev. Lett. 97, 073905, (2006).
[5] J. J.Cook, K. I. Tsakmakidis, and O. Hess, J. Opt. A: Pure Appl. Opt. 11, 114026, (2009).
[6] V. G. Veselago, Sov. Phys. Usp. 10, 509, (1968).
[7] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, Photonic Crystals: Modeling the Flow of Light, Princenton Univ. Press, Princenton, NJ, 1995.
[8] K. Busch, M. Frank, A. Garcia-Martin, D. Hermann, S. F. Mingaleev, M. Schillinger, and L. Tkeshelashvili, Phys. Stat. Sol. (a) 197, 637, (2003).
[9] L.-M. Li and Z.-Q. Zhang, Phys. Rev. B 58, 9587, (1998).
[10] S. Enoch, G. Tayeb, and D. Maystre, Progress In Electromagnetics Research, PIER 41, 61, (2003).
[11] S. Enoch, G. Tayeb, and D. Maystre, Opt. Commun. 161, 171, (1999).
[12] K. Yu. Bliokh and Yu. P. Bliokh, Phys.-Usp. 47, 393, (2004).
[13] A. I. Cabuz, D. Felbacq, and D. Cassagne, Phys. Rev. A **77**, 013807, (2008).