ABSORPTION BAND SHAPE OF COMBINED TWO-DIMENSIONAL MAGNETOEXCITON-CYCLOTRON RESONANCE

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Abstract

The absorption band shape of the combined optical quantum transition with the creation of a two-dimensional magnetoexciton and with the simultaneous excitation of one background electron between its Landau levels is discussed. The combined magnetoexciton-cyclotron resonance (MECR) quantum transitions are described within the frame of the model of two-dimensional electron-hole system in a strong perpendicular magnetic field, taking into account supplementary small concentration of background electrons resident on the lowest Landau level (LLL). The concrete case of magnetoexciton composed by electron and hole on their LLL and the accompanying cyclotron resonance with excitation of the background electron from the LLL to the first excited Landau level is considered. The position of the combined absorption band is shifted in comparison with the frequency of the magnetoexciton band by the frequency of the electron cyclotron resonance. The maximal band width equals the ionization potential I_l of the magnetoexciton, because participation of the third particle side by side with the electron-hole pair permits to uncover its entire internal energy spectrum beginning with the bottom of the magnetoexciton band and finishing with its ionization potential. The analytical formulas describing the absorption band shape in the vicinity of these two limiting frequencies were deduced. The numerical calculations on the base of a general formula permitted us to obtain a full band shape, which has a monotonic decreasing form with a maximal value near the frequency corresponding to the bottom of the magnetoexciton band and tends linearly to zero near the second limiting frequency corresponding to the ionization of magnetoexciton. This band shape is completely different from the case of the combined exciton-cyclotron resonance quantum transitions with participation of the two-dimensional Wannier-Mott exciton in quantum well structures revealed and discussed in references [1, 2].

1. Introduction

The combined quantum transitions with the creation of a two-dimensional Wannier-Mott exciton in the quantum well (QW) structure, accompanied by the simultaneous excitation of one background electron between its Landau levels, were revealed and investigated for the first time in references [1, 2]. The existence of a new-type of three particle optical resonance was demonstrated in QW structure containing two-dimensional electron gas (2DEG) of low density in the presence of an external perpendicular magnetic field. Its strength is not so high to suppose the formation of the magnetoexcitons. It means that the exciton Rydberg constant is greater than the distance between the Landau levels, and the exciton Bohr radius is smaller than the magnetic length l_0. In these conditions an incident photon creates not only an exciton, but, in addition, excites a background electron between its lowest and first excited Landau levels. In Ref. [1, 2] the photoluminescence (PL) spectra, the photoluminescence excitation (PLE) spectra, and reflectivity spectra were used to determine the exciton-cyclotron resonance (ExCR) line. The energy position of this line lies in the range of the Coulomb bound states representing the discrete energy spectrum of the electron-hole (e-h) relative motion of the two-dimensional Wannier-Mott exciton. However, the behavior of the new line in dependence on the magnetic field strength and on the background electron concentration $n_e(p)$ differs greatly from the exciton absorption line. The intensity of the ExCR line increases strongly for higher illumination intensity, i.e., for larger $n_e(p)$, whereas the exciton lines remain insensitive to this factor. Furthermore, the ExCR line shifts linearly with the magnetic field strength with a slope comparable to the electron cyclotron frequency. The ExCR line is strongly σ^- polarized and its Zeeman splitting is similar to that of the 1s heavy hole exciton state. As it was mentioned above, the incident photon absorption creates an exciton and excites a background electron from zeroth to first excited LL.

The theoretical description in Ref. [1, 2] is based on the supposition that the modification of the Wannier-Mott exciton wave functions by magnetic field and by concentration of the background electrons can be neglected. Their spins are parallel to the magnetic field direction. In our description proposed below, in contrast to references [1, 2], we will consider the case of magnetoexcitons, when the distance between the Landau levels is greater than the exciton Rydberg constant, and the magnetic length l_0 is smaller than the exciton Bohr radius. Nevertheless, the suppositions concerning the background electrons will remain the same. Our case can be called combined magnetoexciton-cyclotron resonance (MECR) quantum transition and will be described in the frame of the second order perturbation theory, taking into account the electron-radiation interaction and the Coulomb electron-electron interaction as perturbations.

As the first step of the perturbation theory in our model the incident photon creates a new electron-hole pair. The new electron interacts with the one background electron, giving rise to electron in the final magnetoexciton state and to an electron on the excited Landau level. The electrons being described in the Landau gauge have the states labeled by the quantum number n of Landau quantization and by the unidimensional wave vector p. Together they look as (n, p). Two electrons taking part in the Coulomb scattering process in our concrete case have the initial quantum numbers $(0, f)$ and $(0, h)$, whereas their final quantum numbers are $(1, R)$ and $(0, Q)$. The corresponding matrix element of the Coulomb electron-electron interaction can be denoted as $F_{e-e}(0,f;0,h;1,R;0,Q)$. These types of Coulomb matrix elements were studied in Ref. [3, 4], where their influence on the energy spectrum and collective properties of two-dimensional magnetoexcitons were investigated.

It was shown that the virtual quantum transitions of two interacting Coulomb particles from the lowest Landau levels to excited Landau levels with arbitrary quantum numbers n and m and their transition back to the lowest Landau levels in the second order of the perturbation theory result in indirect attraction between the particles, supplementary to their Coulomb interaction. The influence of this indirect interaction on the chemical potential of the Bose-Einstein condensed magnetoexcitons and on the ground state energy of the metallic-type electron-hole liquid (EHL) was investigated in the Hartree-Fock approximation. The supplementary electron-electron and hole-hole interactions, being averaged with direct pairing of operators, increase the binding energy of magnetoexciton and the energy per pair in the EHL phase. The terms obtained in the exchange pairing of operators give rise to repulsion. Together with the Bogolyubov self-energy terms arising from the electron-hole supplementary interaction, they both influence in favour of BEC of magnetoexcitons with small momentum. The influence of the excited exciton bands on the energy spectrum and on the wave function of the lowest magnetoexciton band was studied in the second order of the perturbation theory.

The knowledge concerning the matrix elements of the Coulomb scattering between Landau levels will be used in our calculations below.

The paper is organized as follows. In section 2, the wave functions of the initial, intermediary, and final states are discussed, and the matrix elements of the perturbation theory in a more general case are considered. In section 3, a simplest case is discussed. In section 4, the absorption band shape is deduced. The conclusions are given in section 5.

2. Combined magnetoexciton – electron quantum transitions

The combined magnetoexciton – electron quantum transitions will be calculated in the second order of the perturbation theory. The Hamiltonian of the electron-radiation interaction in the Faraday geometry was deduced. The light wave vector $\vec{k}$ is oriented along the magnetic field direction. Only the resonant terms are included and only the heavy holes are involved

$$H_{er}=\left(-\frac{e}{m_0}\right)\sum_{\vec{k}(k_x,k_y,k_z)}\sqrt{\frac{2\pi\hbar}{V\omega_k}}\sum_{p,l,l'}\Big\{\Phi\left(l,p;l',p-k_x;k_y\right)\times$$
$$\times P_{cv}\left[C_{\vec{k}\oplus}a^{\dagger}_{l,p,\uparrow}b^{\dagger}_{l',k_x-p,-3/2}+C_{\vec{k}\ominus}a^{\dagger}_{l,p,\downarrow}b^{\dagger}_{l',k_x-p,3/2}\right]+ \tag{1}$$
$$+\Phi\left(l,p;l',p-k_x;-k_y\right)P_{vc}\left[\left(C_{\vec{k}\oplus}\right)^{\dagger}b_{l',k_x-p,-3/2}a_{l,p,\uparrow}+\left(C_{\vec{k}\ominus}\right)^{\dagger}b_{l',k_x-p,3/2}a_{l,p,\downarrow}\right]\Big\},$$

where only the resonant terms are included and only the heavy holes are involved. The following notations were introduced

$$\Phi\left(l,p;l',p-k_x;k_y\right)=\int\varphi_l^*\left(R_y-pl_0^2\right)\varphi_{l'}\left(R_y-\left(p-k_x\right)l_0^2\right)e^{ik_yR_y}dR_y$$
$$C_{\vec{k}\oplus}=C_{k,x}\pm iC_{k,y};\quad P_{vc}=\frac{1}{v_0}\int_{v_0}d\vec{\rho}U^*_{v,0}\left(\vec{\rho}\right)\left(-i\hbar\vec{\nabla}_{\vec{\rho}}\right)U_{c,0}\left(\vec{\rho}\right)\quad . \tag{2}$$

The photon operators $C_{\vec{k}\oplus}$ stand into (1) near the circular polarizations $\sigma_k^{\mp}=\left(\vec{e}_{kx}\mp i\vec{e}_{ky}\right)1/\sqrt{2}$ because

$$\vec{e}_{kx}C_{kx}+\vec{e}_{ky}C_{ky}=\sigma_k^{+}C_{k\ominus}+\sigma_k^{-}C_{k\oplus}.$$

The coefficients $\Phi\left(l;l'\right)$ are expressed through the wave functions of electrons in the strong perpendicular magnetic field

$$\Psi_{n,p}\left(\vec{R}\right)=\frac{e^{ipR_x}}{\sqrt{L}}\varphi_n\left(R_y-pl_0^2\right).$$

In the Landau gauge they are characterized by the quantum number n of Landau quantization in one in-plane direction and by the unidimensional wave vector p in the perpendicular in-plane direction.

The Coulomb e-e interaction describing the quantum transitions of two electrons from the lowest Landau levels (LLLs) to the excited LLs with the numbers *n*` and *m*` has the form

$$H_{Coul} = \frac{1}{2}\sum_{p'q's'}\sum_{n'm'} F_{e-e}\left(0,p';0,q';n',p'-s';m',q'+s'\right) a^{\dagger}_{0,p',\downarrow} a^{\dagger}_{0,q',\downarrow} a_{m',q'+s',\downarrow} a_{n',p'-s',\downarrow} + h.c., \quad (3)$$

where

$$\begin{aligned} &F_{e-e}\left(0,p';0,q';n',p'-s';m',q'+s'\right) = \\ &= \int\int \Psi^{*}_{0,p'}\left(\vec{R}_1\right)\Psi_{n',p'-s'}\left(\vec{R}_1\right) V_{12} \Psi^{*}_{0,q'}\left(\vec{R}_2\right)\Psi_{m',q'+s'}\left(\vec{R}_2\right) d\vec{R}_1 d\vec{R}_2 \end{aligned} \quad (4)$$

and

$$V_{12} = \frac{1}{\varepsilon_0\left|\vec{R}_1 - \vec{R}_2\right|} = \sum_{\vec{\kappa}(\kappa_x,\kappa_y)} e^{i\vec{\kappa}\left(\vec{R}_1-\vec{R}_2\right)} V_{\vec{\kappa}} \ . \quad (5)$$

We will discuss the case of quantum transitions when the initial $\left|i\right\rangle$, intermediary $\left|u_1\right\rangle$, and final $\left|F\right\rangle$ states of the perturbation theory are

$$\begin{aligned} \left|i\right\rangle &= \left(C_{Q\oplus}\right)^{\dagger} a^{\dagger}_{0T\uparrow}\left|0\right\rangle, \\ \left|u_1\right\rangle &= a^{\dagger}_{0f\uparrow} b^{\dagger}_{-3/2} a^{\dagger}_{0h\uparrow}\left|0\right\rangle, \\ \left|F\right\rangle &= a^{\dagger}_{n,R,\uparrow} \hat{\Psi}^{m,0\dagger}_{ex}\left(\vec{k},\ominus\right)\left|0\right\rangle, \\ \hat{\Psi}^{m,0\dagger}_{ex}\left(\vec{k},\ominus\right) &= \frac{1}{\sqrt{N}}\sum_t e^{-ik_y t l_0^2} a^{\dagger}_{m,\frac{k_x}{2}+t,\uparrow} b^{\dagger}_{\frac{k_x}{2}-t,-3/2}. \end{aligned} \quad (6)$$

Here the spin oriented electrons in direction of the external magnetic field and heavy holes with the projection $j_z = -3/2$ are considered. They can be created using the circularly polarized light in one definite direction denoted by $\sigma_k^{\ominus}$.

In the initial state, one electron is on the LLL *n*=0 with wave vector T, whereas in the final state it has a quantum number *n* and wave number *R*.

The exciton creation operator $\hat{\Psi}^{m,0\dagger}_{ex}\left(\vec{k},\ominus\right)$ is characterized by the quantum numbers (*m*, 0) for electron-hole pair and by circular polarization in a definite direction with the magnetic moment projection $M = -1$.

The energies of the mentioned states are

$$\begin{aligned} E_i &= \hbar\omega_Q + E_g + \frac{1}{2}\hbar\omega_{ce}; \ \omega_{ci} = \frac{eH}{m_i c}, \\ E_{u_1} &= 2E_g + \frac{1}{2}\hbar\omega_{c\mu} + \frac{1}{2}\hbar\omega_{ce}; \ \omega_{c\mu} = \omega_{ce} + \omega_{ch}, \\ E_F &= 2E_g + \left(m + n + \frac{1}{2}\right)\hbar\omega_{ce} + \frac{1}{2}\hbar\omega_{c\mu} - I^{m,0}_{ex}\left(k\right), \\ E^{m,0}_{ex}\left(k\right) &= E_g + m\hbar\omega_{ce} + \frac{1}{2}\hbar\omega_{c\mu} - I^{m,0}_{ex}\left(k\right). \end{aligned} \quad (7)$$

The first order matrix elements $\left\langle i\right|H_{er}\left|u_1\right\rangle$ and $\left\langle u_1\right|H_{Coul}\left|F\right\rangle$ are

$$\langle i|H_{er}|u_1\rangle = \left(-\frac{e}{m_0}\right)\sqrt{\frac{2\pi\hbar}{V\omega_Q}}P_{vc}\left[\Phi\left(0,f;0,f-Q_x;-Q_y\right)\delta_{kr}\left(T,h\right)\times\right.$$
$$\left.\times\delta_{kr}\left(g,Q_x-f\right)-\Phi\left(0,h;0,h-Q_x;-Q_y\right)\delta_{kr}\left(f,T\right)\delta_{kr}\left(g,Q_x-h\right)\right]$$
$$\langle u_1|H_{Coul}|F\rangle = \frac{1}{\sqrt{N}}e^{ik_y\left(\frac{k_x}{2}-g\right)l_0^2}\left[F_{e-e}\left(0,f;0,h;n,R;m,k_x-g\right)\times\right. \tag{8}$$
$$\left.\times\delta_{kr}\left(f-R,k_x-g-h\right)-F_{e-e}\left(0,h;0,f;n,R;m,k_x-g\right)\delta_{kr}\left(h-R,k_x-g-f\right)\right],$$

whereas the second order matrix element is

$$\sum_{u_1}\frac{\langle i|H_{er}|u_1\rangle\langle u_1|H_{Coul}|F\rangle}{E_i-E_{u_1}} = 2A\delta_{kr}\left(T+Q_x,k_x+R\right)\sum_f e^{ik_y\left(\frac{k_x}{2}-Q_x+f\right)l_0^2}\Phi\left(0,f;0,f-Q_x;-Q_y\right)\times \tag{9}$$
$$\times\left[F_{e-e}\left(0,f;0,T;n,R;m,k_x-Q_x+f\right)-F_{e-e}\left(0,T;0,f;n,R;m,k_x-Q_x+f\right)\right],$$

where

$$A = \left(-\frac{e}{m_0}\right)\sqrt{\frac{2\pi\hbar}{V\omega_Q}}\frac{1}{\sqrt{N}}\frac{P_{vc}}{\left(\hbar\omega_Q-E_g-\frac{1}{2}\hbar\omega_{c\mu}\right)}. \tag{10}$$
$$\Phi\left(0,f;0,f-Q_x;-Q_y\right)=e^{-iQ_y f l_0^2}$$

The Fermi golden rule gives the probability of the quantum transitions $P\left(\omega_Q,i,F\right)$

$$P\left(\omega_Q,i,F\right)=\frac{2\pi}{\hbar}\left|\sum_{u_1}\frac{\langle i|H_{er}|u_1\rangle\langle u_1|H_{Coul}|F\rangle}{E_i-E_{u_1}}\right|^2\delta_{kr}\left(E_i-E_F\right). \tag{11}$$

Its sum on the final states can be expressed through the response function $S\left(\omega_Q,i\right)$

$$\sum_{|F\rangle}P\left(\omega_Q,i,F\right)=-\frac{2}{\hbar}\Im m\, S\left(\omega_Q,i\right),$$
$$S\left(\omega_Q,i\right)=\langle i|\hat{H}_{er}\frac{1}{E_i-H_0+i\delta}\hat{H}_{Coul}\frac{1}{E_i-H_0+i\delta}\hat{H}_{Coul}\frac{1}{E_i-H_0+i\delta}\hat{H}_{er}|i\rangle. \tag{12}$$

Now we will take into account that any electron resident on the initial states $\left|0,T,\uparrow\right\rangle$ of the lowest Landau level with filling factor v^2 and concentration $n_e\left(\vec{p}\right)=\frac{v^2}{2\pi l_0^2}$ can take part in the combined quantum transition. Their total number is Nv^2. N and l_0 are determined below. It means we introduce the procedure $v^2\sum_T$. The final states are characterized by the given quantum numbers *n* and *m* and by arbitrary quantum numbers $\vec{k}$ and R. They will be taken into account introducing the summation on $\vec{k}$ and R. As a result, the full probability $W\left(\omega_Q,n,m\right)$ has the form

$$W\left(\omega_Q,n,m\right)=v^2\sum_T\sum_{\vec{k}}\sum_R P\left(\omega_Q,T,\vec{k},R\right)=$$

$$=B_0\frac{1}{NV}\sum_{\vec{k}(k_x,k_y)}\sum_R\sum_{f,g}e^{i(k_y-Q_y)(f-g)l_0^2}\times$$
$$\times\{F_{e-e}(0,f;0,k_x-Q_x+R;n,R;m,k_x-Q_x+f)F^*_{e-e}(0,g;0,k_x-Q_x+R;n,R;m,k_x-Q_x+g)+$$
$$+F_{e-e}(0,k_x-Q_x+R;0,f;n,R;m,k_x-Q_x+f)F^*_{e-e}(0,k_x-Q_x+R;0,g;n,R;m,k_x-Q_x+g)- \quad (13)$$
$$-F_{e-e}(0,f;0,k_x-Q_x+R;n,R;m,k_x-Q_x+f)F^*_{e-e}(0,k_x-Q_x+R;0,g;n,R;m,k_x-Q_x+g)-$$
$$-F_{e-e}(0,k_x-Q_x+R;0,f;n,R;m,k_x-Q_x+f)F^*_{e-e}(0,g;0,k_x-Q_x+R;n,R;m,k_x-Q_x+g)\}\times$$
$$\times\delta\left(\hbar\omega_Q-E_{gap}+I_{ex}^{m,0}(\vec{k})-(n+m)\hbar\omega_{ce}-\frac{1}{2}\hbar\omega_{c\mu}\right),$$

where

$$B_0=\frac{(2\pi)^2\left(\frac{e}{m_0}\right)^2|P_{vc}|^2v^2}{\left(\hbar\omega_Q-E_{gap}-\frac{1}{2}\hbar\omega_{c\mu}\right)^2\omega_Q}. \quad (14)$$

3. The concrete case *n* = 1 and *m* = 0

Below the simplest case $n=1$ and $m=0$ will be considered. It means that the new created exciton is formed by the lowest Landau levels (LLLs) for electron $n_e=0$ and hole $n_h=0$. At the same time, there any free electron lying on the LLL, being excited to the state with $n_e'=1$ only, takes part in the combined quantum transition. In such a way, the calculations of the sums entering into formula (13) will be made in the simplest case $m=0$, $n=1$. For this case the direct and exchange Coulomb matrix elements of electron-electron interaction [3, 4]

$$F_{e-e}(0,f;0,k_x-Q_x+R;1,R;0,k_x-Q_x+f)$$
$$F_{e-e}(0,k_x-Q_x+R;0,f;1,R;0,k_x-Q_x+f) \quad (15)$$

are needed. They are described by the general expression [3, 4]

$$F_{e-e}(0,p;0,q;1,p-s;0,q+s)=\sum_\kappa W_{s,\kappa}e^{i\kappa(p-q-s)l_0^2}\frac{(s+i\kappa)l_0}{\sqrt{2}}. \quad (16)$$

Here the denotations were introduced

$$W_{s,\kappa}=V_{s,\kappa}\exp\left[-\frac{(s^2+\kappa^2)l_0^2}{2}\right];\qquad V_{s,\kappa}=\frac{2\pi e^2}{\varepsilon_0 S\sqrt{s^2+\kappa^2}}. \quad (17)$$

Here ε_0 is the background dielectric constant, S is the layer surface area, and l_0 is the magnetic length $l_0^2=\frac{\hbar c}{eH}$. They determine the manifold degeneracy N of the 2D electron state on the Landau levels and the magnetoexciton ionization potential I_l

$$N=\frac{S}{2\pi l_0^2};\qquad I_l=\frac{e^2}{\varepsilon_0 l_0}\sqrt{\frac{\pi}{2}}. \quad (18)$$

Two sums of the direct and exchange Coulomb matrix elements are

$$F\left(\vec{k}-\vec{Q}_{2D},R\right)=\sum_{f}e^{i\left(k_y-Q_y\right)fl_0^2}F_{e-e}\left(0,f;0,k_x-Q_x+R;1,R;0,k_x-Q_x+f\right) \tag{19}$$

and

$$H\left(\vec{k}-\vec{Q}_{2D},R\right)=\sum_{f}e^{i\left(k_y-Q_y\right)fl_0^2}F_{e-e}\left(0,k_x-Q_x+R;0,f;1,R;0,k_x-Q_x+f\right). \tag{20}$$

They depend on the two projections k_x-Q_x and k_y-Q_y of the 2D wave vector $(\vec{k}-\vec{Q}_{2D})$, where $\vec{Q}_{2D}$ is the 2D projection on the layer of the 3D photon wave vector $\vec{Q}$. In the case of the perpendicular incidence of the light beam on the layer surface the projection $\vec{Q}_{2D}$ is zero.

After some transformations we have found

$$\begin{aligned}F\left(\vec{k}-\vec{Q}_{2D},R\right)&=e^{i\left(k_y-Q_y\right)Rl_0^2}\sum_{t,\kappa}W_{t,\kappa}\exp\left\{i\left[\left(k_y-Q_y\right)t-\kappa\left(k_x-Q_x\right)\right]l_0^2\right\}\frac{\left(t+i\kappa\right)l_0}{\sqrt{2}}=\\&=e^{i\left(k_y-Q_y\right)Rl_0^2}\sum_{\vec{P}}W_{\vec{P}}\exp\left\{i\left[\vec{P}\times\left(\vec{k}-\vec{Q}_{2D}\right)\right]_z l_0^2\right\}\frac{\left(P_x+iP_y\right)l_0}{\sqrt{2}}.\end{aligned} \tag{21}$$

Here the 2D wave vector $\vec{P}$ with components $P_x=t$, $P_y=\kappa$ was introduced.

Introducing the representation [5]

$$e^{izSin\varphi}=J_0\left(z\right)+2\sum_{k=1}^{\infty}J_{2k}\left(z\right)Cos2k\varphi+2i\sum_{k=0}^{\infty}J_{2k+1}\left(z\right)Sin\left(\left(2k+1\right)\varphi\right)$$

In formula (21) we will obtain the final expression

$$F\left(\vec{k}-\vec{Q}_{2D},R\right)=-e^{i\left(k_y-Q_y\right)Rl_0^2}I_l\frac{\left|\vec{k}-\vec{Q}_{2D}\right|l_0}{2\sqrt{2}}e^{-\frac{\left|\vec{k}-\vec{Q}_{2D}\right|^2l_0^2}{2}}{}_1F_1\left(\frac{1}{2},2,\frac{\left|\vec{k}-\vec{Q}_{2D}\right|^2l_0^2}{2}\right). \tag{22}$$

Here ${}_1F_1\left(a,b,x\right)$ is the confluent hypergeometric function.

The matrix element $H\left(\vec{k}-\vec{Q}_{2D},R\right)$ (20) can be written in the form

$$\begin{aligned}H\left(\vec{k}-\vec{Q}_{2D},R\right)&=\sum_{\kappa}\sum_{f}V_{k_x-Q_x,\kappa}\exp\left\{-\frac{\left[\left(k_x-Q_x\right)^2+\kappa^2\right]l_0^2}{2}\right\}\times\\&\times\exp\left[if\left(k_y-Q_y-\kappa\right)l_0^2\right]e^{i\kappa Rl_0^2}\left[\frac{\left(k_x-Q_x\right)+i\kappa}{\sqrt{2}}\right]l_0\end{aligned}, \tag{23}$$

taking into account the equality

$$\sum_{f}\exp\left[if\left(k_y-Q_y-\kappa\right)\right]=N\delta_{kr}\left(\kappa,k_y-Q_y\right), \tag{24}$$

the final expression for (23) is

$$H\left(\vec{k}-\vec{Q}_{2D},R\right)=I_le^{i\left(k_y-Q_y\right)Rl_0^2}\exp\left[-\frac{\left|\vec{k}-\vec{Q}_{2D}\right|^2l_0^2}{2}\right]\frac{\left[\left(k_x-Q_x\right)+i\left(k_y-Q_y\right)\right]l_0}{\left|\vec{k}-\vec{Q}_{2D}\right|l_0\sqrt{\pi}}. \tag{25}$$

Final expressions (22) and (25) permit us to calculate the triple sums encountered in expression (13)

$$\sum_R \sum_f \sum_g e^{i(k_y - Q_y)(f-g)l_0^2} \times$$
$$\times\{F_{e-e}(0,f;0,k_x-Q_x+R;1,R;0,k_x-Q_x+f)F^*_{e-e}(0,g;0,k_x-Q_x+R;1,R;0,k_x-Q_x+g)+$$
$$+F_{e-e}(0,k_x-Q_x+R;0,f;1,R;0,k_x-Q_x+f)F^*_{e-e}(0,k_x-Q_x+R;0,g;1,R;0,k_x-Q_x+g)-$$
$$-F_{e-e}(0,f;0,k_x-Q_x+R;1,R;0,k_x-Q_x+f)F^*_{e-e}(0,k_x-Q_x+R;0,g;1,R;0,k_x-Q_x+g)-$$
$$-F_{e-e}(0,k_x-Q_x+R;0,f;1,R;0,k_x-Q_x+f)F^*_{e-e}(0,g;0,k_x-Q_x+R;1,R;0,k_x-Q_x+g)\}= \quad (26)$$
$$=\sum_R\left\{\left|F\left(\vec{k}-\vec{Q}_{2D},R\right)\right|^2+\left|H\left(\vec{k}-\vec{Q}_{2D},R\right)\right|^2-\right.$$
$$\left.-F\left(\vec{k}-\vec{Q}_{2D},R\right)H^*\left(\vec{k}-\vec{Q}_{2D},R\right)-F^*\left(\vec{k}-\vec{Q}_{2D},R\right)H\left(\vec{k}-\vec{Q}_{2D},R\right)\right\}=NI_l^2\exp\left[-\left|\vec{k}-\vec{Q}_{2D}\right|^2 l_0^2\right]\times$$
$$\times\left\{\frac{1}{\pi}+\frac{\left|\vec{k}-\vec{Q}_{2D}\right|^2 l_0^2}{8}\times\left|{}_1F_1\left(\frac{1}{2},2,\frac{\left|\vec{k}-\vec{Q}_{2D}\right|^2 l_0^2}{2}\right)\right|^2+\frac{(k_x-Q_x)l_0}{\sqrt{2\pi}}\,{}_1F_1\left(\frac{1}{2},2,\frac{\left|\vec{k}-\vec{Q}_{2D}\right|^2 l_0^2}{2}\right)\right\}$$

4. The absorption band shape for the incident light perpendicular to the layer

Below, a special case of the perpendicular incidence of the light on the layer surface will be considered. It means we put $\vec{Q}_{2D}=0$ in formula (26). This expression (26) was multiplied by the δ-function $\delta\left(E_i-E_f\right)$ and summarized on the 2D wave vector $\vec{k}$ using the substitution

$$\sum_{\vec{k}}=\frac{S}{(2\pi)^2}\int_0^\infty kdk\int_0^{2\pi}d\varphi .$$

After this summation the third term in expression (26) proportional to k_x will disappear. The argument of the δ-function can be transformed introducing the dimensionless frequency detuning Δ

$$\Delta=\frac{1}{I_l}\left(\hbar\omega_Q-E_{gap}-\frac{1}{2}\hbar\omega_{c\mu}-\hbar\omega_{ce}\right). \quad (27)$$

The δ-function contains the ionization potential $I_{ex}^{00}(k)$ of the magnetoexciton with $n_e=n_h=0$ in the form

$$\delta\left(E_i-E_f\right)=\frac{1}{I_l}\delta\left(\Delta+\frac{I_{ex}^{00}(k)}{I_l}\right). \quad (28)$$

The exact value of the ionization potential can be expressed through the modified Bessel function $I_0(x)$ [6]

$$\frac{I_{ex}^{00}(k)}{I_l}=e^{-\frac{k^2l_0^2}{4}}I_0\left(\frac{k^2l_0^2}{4}\right)\approx\begin{cases}1-\dfrac{k^2l_0^2}{4}, & kl_0\to 0\\ \sqrt{\dfrac{2}{\pi}}\dfrac{1}{kl_0}, & kl_0\to\infty\end{cases}. \quad (29)$$

The approximate values in two limiting cases show the change from 1 to 0 with the negative quadratic dependence at small values of kl_0 and with hyperbolic decreasing in the limit $kl_0 \to \infty$. It means that the frequency detuning Δ changes in the interval $-1 \le \Delta \le 0$, when kl_0 changes in the interval $0 < kl_0 < \infty$, and that the absorption band shape of the combined quantum transition is confined in this frequency interval.

To determine analytically the band shape, we divide the interval of integration on k in two regions. One of them corresponds to $0 < kl_0 < 1$ and $-1 \le \Delta \le -0.9$ and the second region covers the values $1 < kl_0 < \infty$ and $-0.3 \le \Delta \le 0$.

In these two regions of integration on k, δ-function (28) obtain the concrete forms

$$\delta\left(\Delta + \frac{I_{ex}^{00}(k)}{I_l}\right) \approx \begin{cases} \delta\left(\Delta + 1 - \frac{k^2 l_0^2}{4}\right), & \begin{matrix} 0 < kl_0 < 1 \\ -1 \le \Delta \le -0.9 \end{matrix} \\ \delta\left(\Delta + \sqrt{\frac{2}{\pi}}\frac{1}{kl_0}\right), & \begin{matrix} 1 < kl_0 < \infty \\ -0.3 \le \Delta \le 0 \end{matrix} \end{cases}. \quad (30)$$

In the same intervals for the values kl_0, one can find the analytical approximations for the confluent hypergeometric functions ${}_1F_1(a,b,x)$ [7]

$${}_1F_1\left(\frac{1}{2};2;z\right) = \begin{cases} \left(1 + \frac{z}{8}\right), & 0 < z < 1 \\ \frac{e^z}{\sqrt{\pi} z^{3/2}}, & 1 < z < \infty \end{cases}. \quad (31)$$

Now the absorption band shape $W(\omega_Q,1,0)$ in the full frequency interval $-1 \le \Delta \le 0$ can be represented as follows

$$W(\omega_Q,1,0) = \frac{B_0 I_l}{4\pi l_0^2 L}\int_0^\infty x dx e^{-x^2}\left\{\frac{1}{\pi} + \frac{x^2}{8}\left|{}_1F_1\left(\frac{1}{2},2,\frac{x^2}{2}\right)\right|^2\right\}\delta\left(\Delta + e^{-\frac{x^2}{4}} I_0\left(\frac{x^2}{4}\right)\right); \; -1 \le \Delta \le 0. \quad (32)$$

Here L is obtained supposing $V = SL$ and reflects the fact that the light does not undergo the size quantization. If the 2D layer is embedded into the microcavity, in this case L has a finite value equal to the distance between the mirrors. Integral (32) can be simplified in two limiting regions of the frequency detuning Δ, namely, in the vicinity of the value $\Delta = -1$, when $-1 \le \Delta \le -0.9$ and in the vicinity of the point $\Delta = 0$, when $-0.3 \le \Delta \le 0$.

In these two regions on the base of (30) we can write

$$\int_0^\infty x dx e^{-x^2}\left\{\frac{1}{\pi} + \frac{x^2}{8}\left|{}_1F_1\left(\frac{1}{2},2,\frac{x^2}{2}\right)\right|^2\right\}\delta\left(\Delta + e^{-\frac{x^2}{4}} I_0\left(\frac{x^2}{4}\right)\right) =$$

$$= \begin{cases} \frac{1}{2}\int_0^1 dy e^{-y}\left[\frac{1}{\pi} + \frac{y}{8}\left(1 + \frac{y}{8}\right)\right]\delta\left(\Delta + 1 - \frac{y}{4}\right), & -1 \le \Delta \le -0.9 \\ \int_1^\infty x dx e^{-x^2}\left[\frac{1}{\pi} + \frac{e^{x^2}}{\pi x^4}\right]\delta\left(\Delta + \sqrt{\frac{2}{\pi}}\frac{1}{x}\right), & -0.3 \le \Delta \le 0 \end{cases}. \quad (33)$$

The band shape in the interval $-1 \le \Delta \le -0.9$ will be obtained taking the integrand in the point $y = (1+\Delta)4$, whereas in the second region $-0.3 \le \Delta \le 0$ the integrand will be taken in the point $x=-\sqrt{\frac{2}{\pi}}\frac{1}{\Delta}$. Taking into account that $\delta\left(f\left(x\right)\right)=\frac{\delta\left(x-a\right)}{\left|f'\left(x\right)\right|_{x=a}}$ in our cases we will write

$$\delta\left(\Delta+1-\frac{y}{4}\right)=4\delta\left(y-4\left(1+\Delta\right)\right)$$
$$\delta\left(\Delta+\sqrt{\frac{2}{\pi}}\frac{1}{x}\right)=\frac{\sqrt{\frac{2}{\pi}}\delta\left(x+\sqrt{\frac{2}{\pi}}\frac{1}{\Delta}\right)}{\Delta^2}, \tag{34}$$

what gives the final band shape consisting of two parts

$$\frac{W\left(\omega_Q,1,0\right)}{B_0I_l/L2\pi l_0^2}=\begin{cases}e^{-4(1+\Delta)}\left[\frac{2}{\pi}+\left(1+\Delta\right)+\frac{\left(1+\Delta\right)^2}{2}\right], & -1\le\Delta\le-0.9\\ -\frac{2}{\pi}\frac{e^{-\frac{2}{\pi\Delta^2}}}{\Delta^3}-\frac{\Delta}{2}, & -0.3\le\Delta\le 0\end{cases}. \tag{35}$$

In the intermediary region $-0.9\le\Delta\le-0.3$ the absorption band shape is a monotonically decreasing function as was verified by the numerical calculations on the base of exact formula (32).

The plot of the band shape is represented in Fig. 1.

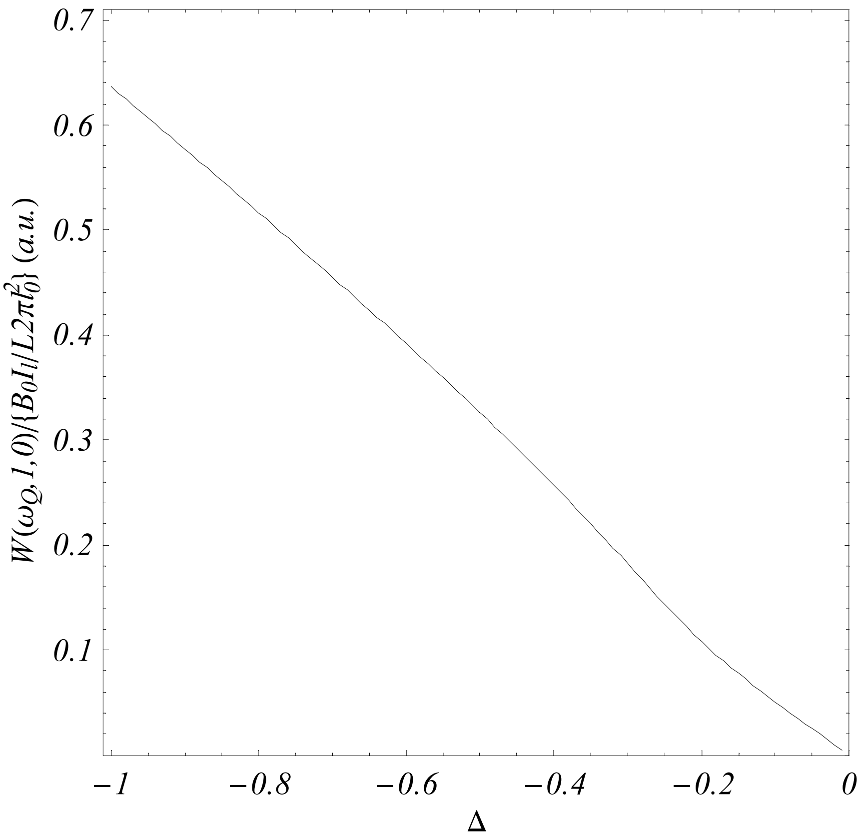

Fig. 1. Absorption band shape of the combined MECR quantum transition in dependence on the dimensionless frequency detuning Δ.

5. Conclusions

The combined magnetoexciton-cyclotron resonance quantum transition was considered in the case when the magnetoexciton is composed by the electron and hole on their lowest Landau levels and the background electron takes part simultaneously in the quantum transition from its lowest to first excited Landau level. The absorption band is situated on the energy scale in the position shifted in comparison with the frequency of the magnetoexciton line by the energy of the electron cyclotron resonance. The band shape has a width equal to the magnetoexciton ionization potential. It begins with the frequency corresponding to the combined transition with the creation of the magnetoexciton at the bottom of its band and finishes at the frequency corresponding to the combined transition with the ionization of the magnetoexciton. The revealing of the internal energy structure of the magnetoexciton became possible due to participation of the background electron in the quantum transition. The analytical formulas describing the absorption band shape in the vicinity of two limiting frequencies mentioned above were deduced. The numerical calculations on the base of a general formula were carried out permitting us to draw the full absorption band shape. It has a form with maximal value in the lower limiting frequency, monotonously decreases and tends linearly to zero near the upper limiting frequency.

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