NONLINEAR EXCITATIONS IN CARBON NANOTUBE

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Abstract

The possibility of realization of both the cubic and the other power-mode and polynomial types of non-linearity in the Shroedinger equation that describes the quantum excitations for n-dimensional media is shown. This problem is first considered in detail for such system as the single-walled carbon nanotube. The stable solitons are obtained analytically.

1. Introduction

During the electronic excitations in media with different degree of order, relatively stable formations such as solitons appear [1-2]. In contrast to the related excitations of excitons, solitons are more real objects. This is connected with the fact that the excitons leave out of account the influence of electronic excitation on the change in interaction between the structural elements of the medium. A similar change leads to the local deformation inside the lattice. A study of soliton excitations in the novel materials such as the single-walled carbon nanotubes (SWCNT) [3-5] is issue of the day, especially as they are connected also with configurational (conformational) changes in the nanotubes. A change in their form can lead to the appearance of heterojunctions [6-7], which are very important to nanoelectronics.

2. Functional for the electronic excitations

Depending on symmetry, the SWCNT can be metals or semiconductors [2-3]. Therefore the corresponding functional for the electronic excitations in the semiconducting SWCNT takes the form:

$$H(\{A\}) = \sum_{\bar{n},\bar{m}}{}' \left(w_{\bar{n},\bar{m}} + D_{\bar{n},\bar{m}} \cdot \left| A_{\bar{n},\bar{m}} \right|^2 + \sum_{\bar{k},\bar{l}}{}' M_{\bar{k},\bar{l}} \cdot A^*_{\bar{n},\bar{m}} \cdot A_{\bar{n}+\bar{k},\bar{m}+\bar{l}} \right). \quad (1)$$

Here $\bar{n}, \bar{m}, \bar{k}, \bar{l}$ are the vectors, which indicate the positions of carbon atoms in the nanotube (vectors of lattice). Primes in sum on $\bar{n}, \bar{m}$ mean that $\bar{n} \neq \bar{m}$, and in sum on $\bar{k}, \bar{l}$ mean that these vectors are not equal to zero simultaneously. Factor $w_{\bar{n},\bar{m}}$ describes the pair interaction between the structural elements of the considered space (here carbon atoms) in the absence of excitation ($A_{\bar{n},\bar{m}} = 0$). Factor $D_{\bar{n},\bar{m}}$ describes changes in interaction between carbon atoms in the presence of excitation ($A_{\bar{n},\bar{m}} \neq 0$) and fixed locations of charge carriers (electron in the knot $\bar{n}$ and hole in the knot $\bar{m}$). Factor $M_{\bar{k},\bar{l}}$ represents the additional change in interaction between carbon atoms, but connected with the movement of carriers in the space. $A_{\bar{n},\bar{m}}$ is the wave function which depends on the variables $\bar{n}$ and $\bar{m}$, and determines the distribution of the excitation of the system in the space (nanotube) and in the time. It satisfies the normalization condition:

$$\sum_{\bar{n},\bar{m}} \left| A_{\bar{n},\bar{m}} \right|^2 = 1 . \qquad (2)$$

Assuming the considered space to be uniform (in this case it indicates the absence of impurities or other defects), it is possible to consider that all enumerated factors in functional (1) depend only on differences in the pairs of their indices. Then functional (1) is reduced to the form

$$H(\{A\}) = \sum_{\bar{N}} \sum_{\bar{n}(\neq 0)} \left(w_{\bar{n}} + D_{\bar{n}} \cdot \left| A_{\bar{N},\bar{n}} \right|^2 + \sum_{\bar{L},\bar{k}}{}' M_{\bar{L},\bar{k}} \cdot A^*_{\bar{N},\bar{n}} \cdot A_{\bar{N}+\bar{L},\bar{n}+\bar{k}} \right), \qquad (3)$$

where $\bar{N} = \bar{n} + \bar{m}$ is the doubled value of the geometric center between the carriers (for the difference $\bar{n} - \bar{m}$ is made the replacement simply on $\bar{n}$). Analogously, for the indices $\bar{k}$ and $\bar{l}$: $\bar{L} = \bar{k} + \bar{l}$, and $\bar{k} - \bar{l}$ we have substituted simply on $\bar{k}$. Prime in the sign of sum on $\bar{L}, \bar{k}$ means that these indices cannot be equal to zero simultaneously. If we temporarily designate equilibrium values of variables $\bar{n}, \bar{N}, \bar{k}, \bar{L}$ in the absence of excitation ($A_{\bar{N},\bar{n}} = 0$) through $\bar{n}_0, \bar{N}_0, \bar{k}_0, \bar{L}_0$, then in the presence of excitation ($A_{\bar{N},\bar{n}} \neq 0$) functional (3) will take the form:

$$H(\{A\}) = \sum_{\bar{N}_0} \sum_{\bar{n}_0(\neq 0)} \left(w_{\bar{n}_0 + \bar{\xi}_{\bar{n}_0,\bar{N}_0}} + D_{\bar{n}_0 + \bar{\xi}_{\bar{n}_0,\bar{N}_0}} \left| A_{\bar{N}_0,\bar{n}_0} \right|^2 + \right. \qquad (4)$$

$$\left. \sum_{\bar{L}_0,\bar{k}_0}{}' M_{\bar{L}_0,\bar{k}_0} A^*_{\bar{N}_0,\bar{n}_0} A_{\bar{N}_0+\bar{L}_0,\bar{n}_0+\bar{k}_0} \right).$$

Additional displacements $\bar{\xi}_{\bar{n}_0,\bar{N}_0}$ are determined by the presence of excitation. Expanding w and D in series on the components of small displacements $\xi^{\alpha}_{\bar{n}_0,\bar{N}_0}$ (here α=1,2), and also by rejecting then indices "0" at the variables $\bar{n}, \bar{N}, \bar{k}, \bar{L}$ (for the purpose of simplification in the notation), we will obtain:

$$w_{\bar{n}+\bar{\xi}_{\bar{n},\bar{N}}} = w_{\bar{n}} + \sum_{\alpha,\beta} U^{\alpha,\beta}_{\bar{n}} \cdot \xi^{\alpha}_{\bar{n},\bar{N}} \cdot \xi^{\beta}_{\bar{n},\bar{N}}, \qquad (5)$$

$$D_{\bar{n}+\bar{\xi}_{\bar{n},\bar{N}}} = D_{\bar{n}} + \sum_{\alpha} D^{\alpha}_{\bar{n}} \cdot \xi^{\alpha}_{\bar{n},\bar{N}} . \qquad (6)$$

Here $U^{\alpha,\beta}_{\bar{n}}$ and $D^{\alpha}_{\bar{n}}$ are the corresponding force "constants", which are, generally speaking, functions from the distance $\bar{n}$ between the carriers. Substituting (5) and (6) in (4) and minimizing the obtained expression on $\xi^{\alpha}_{\bar{n},\bar{N}}$, we will obtain

$$\xi^{\alpha}_{\bar{n},\bar{N}} = -\frac{1}{2}\sum_{\beta} \tilde{U}^{\alpha,\beta}_{\bar{n}}\, D^{\beta}_{\bar{n}} \left| A_{\bar{N},\bar{n}} \right|^2, \tag{7}$$

where $\tilde{U}^{\alpha,\beta}_{\bar{n}}$ is the matrix, reverse with respect to $U^{\alpha,\beta}_{\bar{n}}$ on the superscripts. Substitution of (7) in the functional, which was minimized on $\xi^{\alpha}_{\bar{n},\bar{N}}$ (obtained of (4) by substitutions (5), (6)) gives the form

$$H(\{A\}) = W + \sum_{\bar{N}}\sum_{\bar{n}(\neq 0)}\left(D_{\bar{n}}\left|A_{\bar{N},\bar{n}}\right|^2 - \frac{1}{2}G_{\bar{n}}\left|A_{\bar{N},\bar{n}}\right|^4 + \sum_{\bar{L},\bar{k}}{}' M_{\bar{L},\bar{k}}\, A^{*}_{\bar{N},\bar{n}}\, A_{\bar{N}+\bar{L},\bar{n}+\bar{k}} \right), \tag{8}$$

where $W \equiv \sum_{\bar{N}}\sum_{\bar{n}(\neq 0)} w_{\bar{n}}$ is the energy of the not excited medium and $G_{\bar{n}} = \frac{1}{2}\sum_{\alpha,\beta}\tilde{U}^{\alpha,\beta}_{\bar{n}}\, D^{\alpha}_{\bar{n}}\, D^{\beta}_{\bar{n}}$. In expression (6) it is possible to consider the following terms of expansion on $\xi^{\alpha}_{\bar{n},\bar{N}}$

$$D_{\bar{n}+\bar{\xi}_{\bar{n},\bar{N}}} = D_{\bar{n}} + \sum_{\alpha} D^{\alpha}_{\bar{n}} \cdot \xi^{\alpha}_{\bar{n},\bar{N}} + \sum_{\alpha,\beta} D^{\alpha,\beta}_{\bar{n}} \cdot \xi^{\alpha}_{\bar{n},\bar{N}} \cdot \xi^{\beta}_{\bar{n},\bar{N}}, \tag{9}$$

where $D^{\alpha,\beta}_{\bar{n}}$ is the force "constant", which makes sense, analogous to $U^{\alpha,\beta}_{\bar{n}}$ "constant". For example, this can be important to maximal precision of the contributions of the second order on $\xi^{\alpha}_{\bar{n},\bar{N}}$. Under the condition $D^{\alpha,\beta}_{\bar{n}} << U^{\alpha,\beta}_{\bar{n}}$, the corresponding minimization leads to the following degree on $\left|A_{\bar{N},\bar{n}}\right|^2$

$$\begin{aligned} H(\{A\}) = W + \sum_{\bar{N}}\sum_{\bar{n}(\neq 0)}\Bigg(& D_{\bar{n}}\left|A_{\bar{N},\bar{n}}\right|^2 - \frac{1}{2}G_{\bar{n}}\left|A_{\bar{N},\bar{n}}\right|^4 + \frac{1}{3}S_{\bar{n}}\left|A_{\bar{N},\bar{n}}\right|^6 + \\ & \sum_{\bar{L},\bar{k}}{}' M_{\bar{L},\bar{k}} A^{*}_{\bar{N},\bar{n}} A_{\bar{N}+\bar{L},\bar{n}+\bar{k}} \Bigg), \end{aligned} \tag{10}$$

where $S_{\bar{n}} = \sum_{\alpha,\beta} 3D^{\alpha,\beta}_{\bar{n}} g^{\alpha}_{\bar{n}}\, g^{\beta}_{\bar{n}}$ and $g^{\alpha}_{\bar{n}} = \sum_{\beta}\frac{1}{2}\tilde{U}^{\alpha,\beta}_{\bar{n}} D^{\beta}_{\bar{n}}$. We will further work with this functional, since functional (8) is one of its special cases.

For final obtaining of a strict functional of soliton let us remind that in the sum on $\bar{L}$ and $\bar{k}$ these indices are not equal to zero simultaneously. Choosing separately $\bar{L} = 0,\ \bar{k} \neq 0$ terms, and $\bar{L} \neq 0,\ \bar{k} = 0$ terms, and rejecting all the remaining as negligible, and also representing the function $A_{\bar{N},\bar{n}}$ in the form of product $A_{\bar{N},\bar{n}} = \varphi_{\bar{N}}\,\psi_{\bar{n}}$, transforms the considered functional transforms to the form:

$$H(\{\varphi\},\{\psi\}) = W + \sum_{\overline{N}} \left(\left|\varphi_{\overline{N}}\right|^2 \sum_{\overline{n}\neq 0} \left[\sum_{\overline{k}\neq 0} M_{o,\overline{k}}\, \psi^*_{\overline{n}}\, \psi_{\overline{n}+\overline{k}} + D_{\overline{n}} \left|\psi_{\overline{n}}\right|^2 \right] + \sum_{\overline{L}} M_{\overline{L},0}\, \varphi^*_{\overline{N}}\, \varphi_{\overline{N}+\overline{L}} \right) - \sum_{\overline{N}} \left(\frac{1}{2} \left|\varphi_{\overline{N}}\right|^4 \sum_{\overline{n}} G_{\overline{n}} \left|\psi_{\overline{n}}\right|^4 - \frac{1}{3} \left|\varphi_{\overline{N}}\right|^6 \sum_{\overline{n}} S_{\overline{n}} \left|\psi_{\overline{n}}\right|^6 \right). \quad (11)$$

In this functional, in the term with the sum on $\overline{L}$, the normalization condition is taken into account

$$\sum_{\overline{n}} \left|\psi_{\overline{n}}\right|^2 = 1, \quad (12)$$

which follows from condition (2). Since the function $\psi_{\overline{n}}$ describes the internal state of the system of the excited carriers (electron and hole), and factor $D_{\overline{n}}$ actually is reduced to screened Coulomb interaction between them, we can consider the function $\psi_{\overline{n}}$ as eigenfunction relative to the equation

$$\sum_{\overline{k}\neq 0} M_{o,\overline{k}}\, \psi_{\overline{n}+\overline{k}} + D_{\overline{n}}\, \psi_{\overline{n}} = \epsilon\, \psi_{\overline{n}}, \quad (13)$$

where ϵ is the eigenvalue of energy. It is not difficult to obtain, in accordance with (12) and (13), $\sum_{\overline{n}\neq 0} \left(\sum_{\overline{k}\neq 0} M_{o,\overline{k}}\, \psi^*_{\overline{n}}\, \psi_{\overline{n}+\overline{k}} + D_{\overline{n}} \left|\psi_{\overline{n}}\right|^2 \right) = \epsilon$. Taking into account this, and also the normalization condition

$$\sum_{\overline{N}} \left|\varphi_{\overline{N}}\right|^2 = 1, \quad (14)$$

we will obtain soliton functional in the common form

$$H(\{\varphi\}) = W + \epsilon + \sum_{\overline{N}} \left(\varphi^*_{\overline{N}} \sum_{\overline{L}\neq 0} M_{\overline{L},0}\, \varphi_{\overline{N}+\overline{L}} - \frac{1}{2} g \left|\varphi_{\overline{N}}\right|^4 + \frac{1}{3} \sigma \left|\varphi_{\overline{N}}\right|^6 \right), \quad (15)$$

where it is marked: $g = \sum_{\overline{n}} G_{\overline{n}} \left|\psi_{\overline{n}}\right|^4$ and $\sigma = \sum_{\overline{n}} S_{\overline{n}} \left|\psi_{\overline{n}}\right|^6$.

3. Soliton equation and its solutions

Functional (15) is considered as the Hamiltonian system with the pairs of the canonically conjugated variables $\varphi_{\overline{N}}$ and $p_{\overline{N}} = i\hbar\, \varphi^*_{\overline{N}}$, to which it is possible to apply Hamilton's equations:

$$\frac{\partial \varphi_{\overline{N}}}{\partial t} = \frac{\partial H}{\partial p_{\overline{N}}} \equiv \frac{1}{i\hbar} \frac{\partial H}{\partial \varphi^*_{\overline{N}}}; \qquad \frac{\partial p_{\overline{N}}}{\partial t} \equiv i\hbar \frac{\partial \varphi^*_{\overline{N}}}{\partial t} = -\frac{\partial H}{\partial \varphi_{\overline{N}}}. \quad (16)$$

Substituting in these equations Hamiltonian (15) and taking into account that $M_{\overline{L},0} \equiv M_{-\overline{L},0}$ (since this value depends on $\left|\overline{L}\right|$) we will obtain two complexly conjugated equations. Therefore it is quite enough to consider only one of them:

$$i\hbar\frac{\partial\varphi_{\bar{N}}}{\partial t}=\sum_{\bar{L}>0}M_{\bar{L},0}\cdot\left(\varphi_{\bar{N}+\bar{L}}+\varphi_{\bar{N}-\bar{L}}\right)-g\left|\varphi_{\bar{N}}\right|^{2}\varphi_{\bar{N}}+\sigma\left|\varphi_{\bar{N}}\right|^{4}\varphi_{\bar{N}}. \quad (17)$$

Generally speaking, this is a soliton equation, formulated in the finite differences. But it is more known in the continuous approximation, when set $\bar{N}\equiv\bar{r}$, where $\bar{r}$ is the vector, which takes the continuum of values. This transition is accomplished by the expansion

$$\sum_{\bar{L}>0}M_{\bar{L},0}\cdot\left(\varphi_{\bar{N}+\bar{L}}+\varphi_{\bar{N}-\bar{L}}\right)\Rightarrow\sum_{\bar{L}>0}M_{\bar{L},0}\cdot\left(2\varphi(\bar{r})+\left|\bar{L}\cdot\nabla\right|^{2}\varphi(\bar{r})\right)\equiv$$
$$M\varphi(\bar{r})+\sum_{\alpha}\sum_{L_{\alpha}>0}\mu_{L_{\alpha}}L_{\alpha}^{2}\frac{\partial^{2}\varphi(\bar{r})}{\partial x_{\alpha}^{2}}, \quad (18)$$

where $M\equiv\sum_{\bar{L}>0}2M_{\bar{L},0}$, and $\mu_{L_{\alpha}}$ is reduced to the following identity in the two-dimensional case: $\mu_{L_{\alpha}}\equiv\sum_{L_{\beta}}M_{L_{\alpha},L_{\beta};0}\quad(\alpha\neq\beta)$. It is not difficult to see that here it is possible to introduce the effective mass tensor: $\frac{\hbar^{2}}{2m_{\alpha}}\equiv-\sum_{L_{\alpha}>0}\mu_{L_{\alpha}}L_{\alpha}^{2}$. Then equation (17) in the continuous approximation is reduced to the following:

$$i\hbar\frac{\partial\varphi}{\partial t}+\sum_{\alpha}\frac{\hbar^{2}}{2m_{\alpha}}\frac{\partial^{2}\varphi}{\partial x_{\alpha}^{2}}-M\varphi+g\left|\varphi\right|^{2}\varphi-\sigma\left|\varphi\right|^{4}\varphi=0. \quad (19)$$

In certain cases the symmetry of matrix elements $M_{\bar{L},0}$ is such that the effective mass tensor becomes isotropic. But even if not, then, in the last equation, always (if M, g and s are constants) it is possible to carry out scale conversion, so that it is reduced to the classical soliton equation:

$$i\hbar\frac{\partial\varphi}{\partial t}+\frac{\hbar^{2}}{2m}\Delta\varphi-M\varphi+g\left|\varphi\right|^{2}\varphi-\sigma\left|\varphi\right|^{4}\varphi=0. \quad (20)$$

By simple amplitude-scale conversions it is possible to transform this equation to

$$i\frac{\partial\Phi}{\partial\tau}+\frac{\partial^{2}\Phi}{\partial x^{2}}+\frac{\partial^{2}\Phi}{\partial y^{2}}+f_{0}\Phi+f_{1}\left|\Phi\right|^{2}\Phi+f_{2}\left|\Phi\right|^{4}\Phi=0, \quad (21)$$

where variables x and y are described by the dimensionless quantities in the units of the dimensional parameter: $x_{0}=\frac{\hbar}{|g|}\cdot\sqrt{\frac{|\sigma|}{2m}}$. All other values Φ, τ, f_0, f_1 and f_2 are determined by the specific relations: $\Phi=A\cdot\varphi$; $t=\tau\cdot t_{0}$; $A=\sqrt{\frac{|g|}{|\sigma|}}$; $t_{0}=\hbar\cdot\frac{|\sigma|}{|g|^{2}}$; $f_{0}=-\frac{M\cdot|\sigma|}{|g|^{2}}$; $f_{1}=\mathrm{sign}(g)$; $f_{2}=-\mathrm{sign}(\sigma)$. Hyperbolic type generalized solitons first of all are here realized. They can be attributed to the traditional solutions. They depend on the variable of common form $z=a\cdot x+b\cdot y+c$ and are localized in the direction perpendicular to $a\cdot x+b\cdot y+c=\mathrm{const}$ line. These solutions take the form:

$$\Phi(\tau, z) = B \cdot \frac{\exp(i \cdot (q \cdot z - \omega \cdot \tau))}{\sqrt{1 + \beta \cdot ch(\alpha \cdot (z - 2q\tau))}}. \tag{22}$$

Without the limitation of generality variable z lets us select in the simplest form: $z = \frac{x+y}{\sqrt{2}}$. Then by using solution (22) in equation (21) we obtain the following relations for the parameters B, q, ω, β and α, in which this equation becomes identity: $\omega = q^2 - f_0 - \frac{\alpha^2}{4}$; $\beta = \sqrt{1 + \frac{4}{3}\frac{f_2}{f_1^2} \cdot \alpha^2}$; $B^2 = \frac{\alpha^2}{f_1}$. As one can see, there are three these relations for five parameters. In this case the relations have a form of dependence on the parameters B, β, and ω on the parameter α. Furthermore, the parameter ω depends also on the parameter q (so-called dispersion dependence). The parameter α is determined by the normalization condition (14), and is equal to:

$$\alpha = \pm \frac{f_1 \cdot th\left(\frac{1}{2}\sqrt{-\frac{f_2}{3}}\right)}{\sqrt{-\frac{f_2}{3}} \cdot \left(1 + th^2\left(\frac{1}{2}\sqrt{-\frac{f_2}{3}}\right)\right)}. \tag{23}$$

From the condition $B^2 = \frac{\alpha^2}{f_1}$ follows that the considered type of solitons is realized only in the case of $f_1 > 0$, i.e. actually at $f_1 = 1$ (taking into account the following determination: $f_1 = sign(g)$). This means that hyperbolic type solitons are characteristic only of the systems, which have $g > 0$. In this case σ can have any sign, since the parameter f_2 can take value both 1 and -1.

Besides this solution Lorenz type solitons are realized. The character of these solitons is localized in all directions

$$\varphi(\tau, \vec{r}) = B \cdot \frac{\exp\left(i \cdot \left(\vec{q} \cdot \vec{r} - \omega \cdot \tau\right)\right)}{\sqrt{1 + \beta \cdot \left|\vec{r} - 2 \cdot \tau \cdot \vec{q}\right|^2}}, \tag{24}$$

where $\vec{q} = (q_1, q_2)$ and $\vec{r} = (x,y)$. In this case for the parameters B, $\vec{q}$, β and ω we obtain also three relations

$$\omega(q) = q^2 - f_0; \qquad \beta = 3 \cdot \frac{f_1^2}{f_2}; \qquad B^2 = -3 \cdot \frac{f_1}{f_2}, \tag{25}$$

where it is marked: $q^2 \equiv |\vec{q}|^2 \equiv q_1^2 + q_2^2$. Here we have $f_1 = -1$ and $f_2 = 1$, i.e. in this case $g < 0$, $\sigma < 0$. However this solution does not satisfy the normalization condition (14). But in this case it is important that condition (2) will be executable. It is possible due to satisfaction of condition (12). Therefore, for the SWCNT, Lorenz solitons, presented in Fig.1, can also be of significant interest.

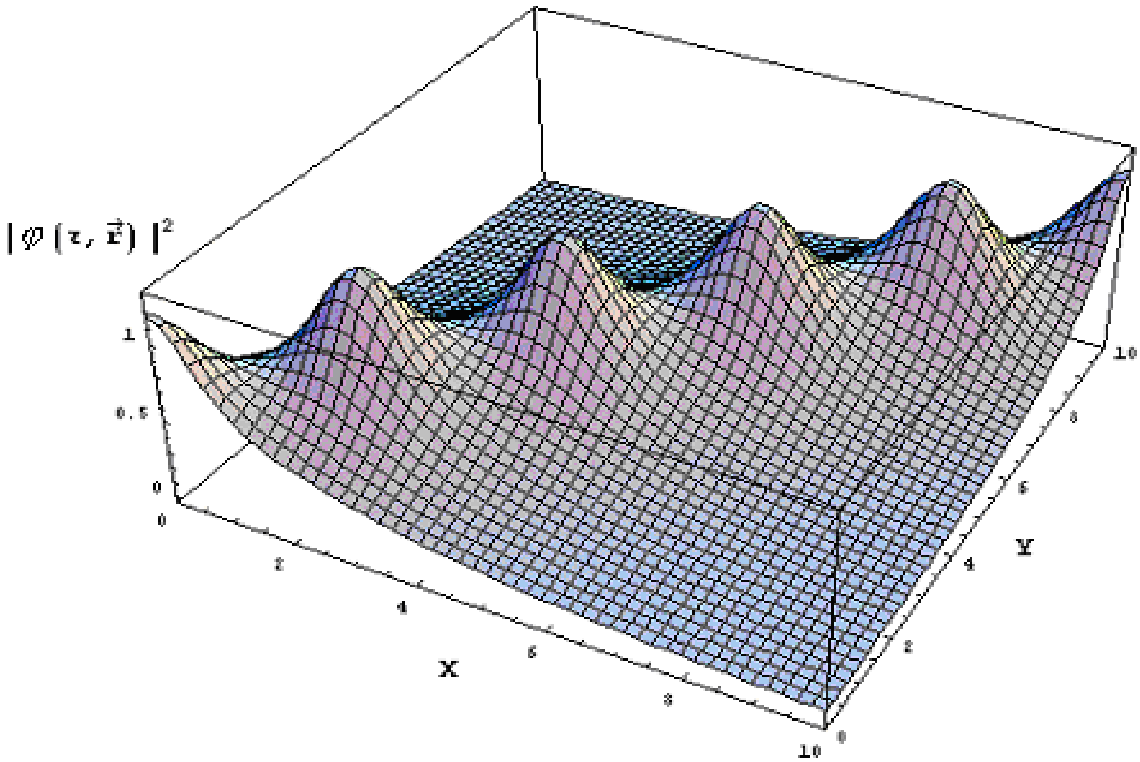

Fig. 1. The image of six consistent Lorenz solitons in the form of joint surface, which is determined by the $|\varphi(\tau,\vec{r})|^2$ function at the following values of the parameters: $q_1=q_2=\overline{0{,}5}$.

4. Summary

The possibility of the realization of soliton excitations in the single walled carbon nanotubes with two types of space charge carriers (electron and hole) was considered in this paper. It is shown that the type of the realized soliton depends on the force "constants" $U_{\underline{n}}^{\alpha,\beta}$, $D_{\underline{n}}^{\alpha}$, determined in formulas (5), (6), and the force "constant" $D_{\underline{n}}^{\alpha,\beta}$, determined in formula (9). These force "constants" substantially govern the values and the signs of the parameters g and σ, which determine the type of the realized soliton. It should be noted that these force "constants" determine also the nature of a change in the configuration (conformation) of nanotubes. Such changes are caused by the appearance of internal stresses in the nanotube, connected with the displacement vector $\xi^{\alpha}_{\overline{n},\overline{N}}$, determined in (7). In our opinion, the obtained theoretical results can be useful for studying the mechanical and electronic properties of carbon nanotubes.

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References

[1] M.J. Ablowitz, H. Segur, “Solitons and the Invers Scattering Transform”, SIAM, Philadelpia, 1981.
[2] A.D. Suprun, Funct. Mater. 8, 1 (2001).
[3] M.S. Dresselhause, G. Dresselhause, P.C. Eklund, “Science of Fullerenes and Carbon Nanotubes”, Academic Press, New York, 1996.
[4] R. Saito, G. Dresselhause, M.S. Dresselhause, “Physical Properties of Carbon Nanotubes”, Singapore, Imperial College Press, 1998.
[5] P. Scharff, Carbon 36, 481 (1998).
[6] R. Saito, G. Dresselhause, M.S. Dresselhause, Phys.Rev. B 53, 2044 (1996).
[7] Yu.I. Prylutskyy, O.V. Ogloblya, P.C. Eklund, P. Scharff, Synth. Met.121, 1209 (2001).